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Unified poincaré and hardy inequalities with sharp constants for convex domains

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Abstract

Let Ω be an n-dimensional convex domain, and let ν ∈ [0,1/2]. For all f ∈ H0 1(Ω) we prove the inequality ∫Ω |∇ f2 dx ≥ (1/4 - ν2) ∫Ω |f|2/δ2 dx + λν 2/δ0 2 ∫Ω |f|2 dx, where δ = dist(x, ∂Ω), δ0 = sup δ. The factor λν 2, is sharp for all dimensions, λν being the first positive root of the Lamb type equation Jν(λν) + 2λνJ′ ν(λν) = 0 for Bessel's functions. In particular, the case ν = 0 with λ0 = 0,940 . . . presents a new sharp form of the Hardy type inequality due to Brezis and Marcus, while in the case ν = 1/2 with λ1/2 = π/2 we obtain a unified proof of an isoperimetric inequality due to Poincaré for n = 1, Hersch for n = 2 and Payne and Stakgold for n ≥ 3. A generalization, when the latter integral is replaced by the integral ∫Ω |f|2/δ 2-m dx, m > 0, is proved, too. As a special case, we obtain the sharp inequality ∫Ω |∇ f|2 dx ≥ m 2 j1/m-1 2/4δ0 m ∫Ω |f|2/δ2-m dx, where j ν is the first positive zero of Jν. © 2007 WILEY-VCH Verlag GmbH & Co. KGaA,.

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Keywords

Bessel functions, Convex domain, Hardy inequality, Poincaré inequality